CSC 303 Teamwork T09

| Teamworks in this course are to be done collaboratively. In teams, it is important to make sure that everyone is contributing to and benefiting from the discussion. Brainstorming and debating ideas with other students is an excellent way to You are welcome to consult the professor, your teammates, the textbook, and other online resources (***not*** including ones that provide solutions to our specific problems) to complete any teamwork, but you *must* explicitly acknowledge any sources besides the professor, your teammates, and the textbook in the *Acknowledgements* section at the end of the document. |
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This teamwork covers material from Chapter 5, Section(s) 5.1 from [**the textbook**](https://moodle.berea.edu/pluginfile.php/737252/mod_resource/content/1/Sipser_Introduction.to.the.Theory.of.Computation.3E.pdf).

1. Given a graph G =〈V, E〉, we say the graph has a *Hamiltonian Cycle* if there is some path that can be taken through the graph that a) visits every node EXACTLY ONCE, and b) ends at the same node it started on. We say the graph has a *Hamiltonian Path* if there is a path that visits every node exactly one and does not end where it started.

Brainstorm a way to modify the graph G (you can add or delete nodes and/or edges, change them, etc.) into a new graph G’ such that G’ contains a Hamiltonian Path in *exactly the same set of cases* that the original graph G contains a Hamiltonian Cycle.

1. Given a graph G =〈V, E〉, a set of nodes V' is said to be an *Independent Set* if no two nodes in V’ are connected by an edge in E. A set of nodes V' is said to be a *Vertex Cover* if every edge in E has at least one endpoint in V'. These two problems seem very distinct from one another. Spoiler alert: they’re not. Explain how Vertex Cover and Independent Set are related to each other, and why if you solve an instance of one you automatically solve an instance of the other. Also consider the problem of finding a *minimal* vertex cover – that is, the (a) *smallest possible* set of nodes such that every edge has at least one endpoint in the set. If you found the minimal vertex cover of the graph, what does that mean about the independent set that you found in the process?
2. Given a graph G =〈V, E〉, a set of nodes V' is said to be a *clique* if every node in V' is connected to every other node in V' by an edge in E. A boolean formula is said to be in 3-Conjunctive Normal Form (3CNF) if it is of the following format: C1 AND C2 AND … AND Cn, where each clause Ci is a boolean expression (xi OR xj OR xk) – note that there are exactly 3 variables per clause. Such an expression is *satisfiable* if there is some setting of truth values to the variables xi that causes the entire formula to evaluate to true. The problem CLIQUE takes as input a graph G and an integer k, and asks if a clique of size k or greater exists in G – this is a *decision problem*, so the only possible answers are “true” and “false”. The problem 3SAT asks if a given boolean formula in 3CNF is satisfiable. Explain the connection between these two problems. **HINT: if the number k corresponds to the number of clauses in the original 3CNF formula, how could you use that information to construct a graph that has a clique of size at least k EXACTLY WHEN the 3CNF formula it represents is satisfiable?**

| **Acknowledgements:** |
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